Strand IV: Number Sense and Numeration

Standard 1: Concepts and Properties of Numbers - Students experience counting and measuring activities to develop intuitive sense about numbers, develop understanding about properties of numbers, understand the need for the existence of different sets of numbers, and investigate properties of special numbers.

Key Ideas

1. An intuitive quantitative sense develops from students' investigations of numbers and their properties.
2. A solid understanding of the numeration system is essential for later success with calculations.
3. Important properties provide students with deeper insight into numbers and their uses.
4. Numeration systems become most useful as students use them to model and describe problems.

Middle School Benchmark	Grade 5	Grade 6	Grade 7	Grade 8
1. Develop an understanding of integers and rational numbers and represent rational numbers in both fraction and decimal form.	**Understand fractions as division statements; find equivalent fractions** **N.ME.05.10** Understand a fraction as a statement of division, e.g., $2 \div 3 = \frac{2}{3}$, using simple fractions and pictures to represent.	**Represent rational numbers as fractions, or decimals** **N.ME.06.06** Represent rational numbers as fractions or terminating decimals when possible, and translate between these representations. **N.ME.06.07** Understand that a fraction or a negative fraction is a quotient of two integers, e.g., $-\frac{8}{3}$ is -8 divided by 3. **Understand rational numbers and their location on the number line** **N.ME.06.17** Locate negative rational numbers (including integers) on the number line; know that numbers and their negatives add to 0, and are on opposites sides and at equal distance from 0 on a number line. **N.ME.06.18** Understand that rational numbers are quotients of integers (nonzero denominators), e.g., a rational number is either a fraction or a negative fraction.		**Understand real number concept** **N.ME.08.03** Understand that in decimal form, rational numbers either terminate or eventually repeat, and that calculators truncate or round repeating decimals; locate rational numbers on the number line; know fraction forms of common repeating decimals, e.g. $0.\bar{1} = \frac{1}{9}; 0.\bar{3} = \frac{1}{3}$.
2. Extend their understanding of numeration systems to include decimal numeration, scientific numeration and non-decimal numeration systems.		**Use exponents** **N.ME.06.16** Understand and use integer exponents, excluding powers of negative numbers; express numbers in scientific notation.	**Recognize irrational numbers** **N.MR.07.06** Understand the concept of square root and cube root, and estimate using calculators.	**Understand real number concepts** **N.ME.08.01** Understand the meaning of a square root of a number and its connection to the square whose area is the number; understand the meaning of a cube root and its connection to the volume of a cube. **N.ME.08.02** Understand meanings for zero and negative integer exponents.

3. Develop an understanding of the properties of the integer and rational number systems (e.g., order, density) and of the properties of special numbers including 0, 1 and , and the additive and multiplicative inverses.		**Represent rational numbers as fractions, or decimals** **N.ME.06.05** Order rational numbers and place them on the number line. **Add and subtract integers and rational numbers** **N.MR.06.08** Understand integer subtraction as the inverse of integer addition; add and subtract integers, using integers from 10 to -10. **Understand rational numbers and their location on the number line** **N.ME.06.17** Locate negative rational numbers (including integers) on the number line; know that numbers and their negatives add to 0, and are on opposites sides and at equal distance from 0 on a number line. **Understand rational numbers and their location on the number line** **N.ME.06.19** Understand that 0 is an integer that is neither negative nor positive. **N.ME.06.20** Know that the absolute value of a number is the value of the number, ignoring the sign; or is the distance of the number from 0. **Solve equations** **A.FO.06.13** Understand that multiplying or dividing both sides of an equation by the same non-zero number creates a new equation that has the same solutions.	**Apply basic properties of real numbers in algebraic contexts** **A.PA.07.11** Understand and use basic properties of real numbers: additive and multiplicative identities, additive and multiplicative inverses, commutativity, associativity, and the distributive property of multiplication over addition. **Combine algebraic expressions and solve equations** **A.FO.07.12** Add, subtract and multiply simple algebraic expressions of the first degree, e.g., (92x + 8y) – 5x + y, or – 2x (5x – 4), and justify using properties of real numbers.	
4. Apply their understanding of number systems to model and solve mathematical and applied problems.				

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Standard 2: Representation and Uses of Numbers - Students recognize that numbers are used in different ways such as counting, measuring, ordering and estimating, understand and produce multiple representations of a number, and translate among equivalent representations.

Key Ideas:
1. Students recognize and understand numbers that they encounter in varied formats.
2. Numeracy requires that students recognize when numbers are equivalent even though they may be represented in different formats.
3. Numbers are used for varied purposes, and it is important to differentiate among their uses.
4. Estimation is one of the most important skills for students to develop and use on a regular basis.
5. Knowing what numbers to use and how to represent them is key to students' abilities to solve problems.

Middle School Benchmark	Grade 5	Grade 6	Grade 7	Grade 8
1. Give geometric representations of fractions, prime and composite numbers, triangular and square numbers, and other number concepts; represent rational numbers and integers on the number line.	**Understand fractions as division statements; find equivalent fractions** **N.ME.05.10** Understand a fraction as a statement of division, e.g., $2 \div 3 = \frac{2}{3}$, using simple fractions and pictures to represent.	**Represent rational numbers as fractions, or decimals** **N.ME.06.05** Order rational numbers and place them on the number line.		**Understand real number concepts** **N.ME.08.03** Understand that in decimal form, rational numbers either terminate or eventually repeat, and that calculators truncate or round repeating decimals; locate rational numbers on the number line; know fraction forms of common repeating decimals, e.g. $0.\bar{1} = \frac{1}{9}; 0.\bar{3} = \frac{1}{3}$.
2. Recognize equivalent representations of a number, especially fractions, decimals and percents, and translate freely among representations.	**Understand meaning of decimal fractions and percentages** **N.ME.05.08** Understand the relative magnitude of ones, tenths, and hundredths and the relationship of each place value to the place to its right, e.g., 1 is 10 tenths, one tenth is 10 hundredths. **N.ME.05.09** Understand percentages as parts out of 100, use % notation, and express a part of a whole as a percentage. **Understand fractions as division statements; find** **N.ME.05.11** Given two fractions, express them as equivalent fractions with a common denominator, but not necessarily a least common denominator, e.g., $\frac{1}{2} = \frac{4}{8}$ and $\frac{3}{4} = \frac{6}{8}$; use denominators less than 12, or factors of 100. **Express, interpret, and use ratios; find equivalences** **N.MR.05.22** Express fractions and decimals as percentages, and vice versa. **N.ME.05.23** Express ratios in several ways, given applied situations, e.g., 3 cups to 5 people, 3:5, $\frac{3}{5}$; recognize and find equivalent ratios.	**Represent rational numbers as fractions, or decimals** **N.ME.06.06** Represent rational numbers as fractions or terminating decimals when possible, and translate between these representations.		**Understand real number concepts** **N.ME.08.03** Understand that in decimal form, rational numbers either terminate or eventually repeat, and that calculators truncate or round repeating decimals; locate rational numbers on the number line; know fraction forms of common repeating decimals, e.g. $0.\bar{1} = \frac{1}{9}; 0.\bar{3} = \frac{1}{3}$.
3. Distinguish between numbers that are used for counting, numbers that are used for ordering, numbers that are used for measuring and numbers that are used for naming.				

4. Develop and refine strategies for estimating quantities, including fractional quantities, and evaluate the reasonableness and appropriateness of their estimates.	**Understand meaning of decimal fractions and percentages** **N.ME.05.08** Understand the relative magnitude of ones, tenths, and hundredths and the relationship of each place value to the place to its right, e.g., 1 is 10 tenths, one tenth is 10 hundredths. **Solve applied problems with fractions** **N.FL.05.20** Solve applied problems involving fractions and decimals; include rounding of answers and checking reasonableness; use examples involving money.	**Solve decimal, percentage and rational number problems** **N.FL.06.14** For applied situations, estimate the answers to calculations involving operations with rational numbers.	**Recognize irrational numbers** **N.MR.07.06** Understand the concept of square root and cube root, and estimate using calculators. **Compute with rational numbers** **N.FL.07.09** Estimate results of computations with rational numbers.	**Understand real number concepts** **N.ME.08.04** Understand that irrational numbers are those that cannot be expressed as the quotient of two integers, and cannot be represented by terminating or repeating decimals; approximate the position of familiar irrational numbers, e.g. $\sqrt{2}$, $\sqrt{3}$, on the number line. **N.FL.08.05** Estimate and solve problems with square roots and cube roots, using calculators. **N.FL.08.06.** Find square roots of perfect squares and approximate the square roots of non-perfect squares by locating between consecutive integers, e.g. $\sqrt{130}$ is between 11 and 12.
5. Select appropriate representations for numbers, including integers and rational numbers in order to simplify and solve problems.	**Solve applied problems with fractions** **N.FL.05.20** Solve applied problems involving fractions and decimals; include rounding of answers and checking reasonableness; use examples involving money.	**Solve decimal, percentage and rational number problems** **N.FL.06.13** Solve word problems involving percentages in such contexts as sales taxes and tips, and involving positive rational numbers. **N.FL.06.14** For applied situations, estimate the answers to calculations involving operations with rational numbers.	**Understand derived quantities** **N.FL.07.02** Solve problems involving derived quantities. **Understand and solve problems involving rates, ratios, and proportions** **N.FL.07.05** Solve simple proportion problems using such methods as unit rate, scaling, finding equivalent fractions, and solving the proportion equation a/b = c/d; know how to see patterns about proportional situations in tables.	**Understand real number concepts** **N.FL.08.05** Estimate and solve problems with square roots and cube roots, using calculators. **Solve problems** **N.MR.08.08** Solve problems involving percent increases and decreases. **N.FL.08.09** Solve problems involving compounded interest or multiple discounts. **N.MR.08.10** Calculate weighted averages such as course grades, consumer price indices, and sports ratings. **N.MR.08.11** Solve problems involving ratio units, such as miles per hour, dollars per pound, or persons per square mile.

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Standard 3: Number Relationships - Students investigate relationships such as equality, inequality, inverses, factors and multiples, and represent and compare very large and very small numbers.

Key Ideas:
1. Relationships of equality and inequality are among the most fundamental in mathematics.
2. Students learn the importance of making comparisons between numbers, especially as ratios and rates.
3. By classifying numbers according to their properties and identifying important numerical relationships, students develop a deeper understanding of numbers.
4. Numbers that are related exponentially exhibit important relationships that students will encounter in a variety of applications.
5. Students can invoke important number relationships to help them understand and solve problems.

Middle School Benchmark	Grade 5	Grade 6	Grade 7	Grade 8
1. Compare and order integers and rational numbers using relations of equality and inequality.	**Understand meaning of decimal fractions and percentages** **N.ME.05.08** Understand the relative magnitude of ones, tenths, and hundredths and the relationship of each place value to the place to its right, e.g., 1 is 10 tenths, one tenth is 10 hundredths.	**Represent rational numbers as fractions, or decimals** **N.ME.06.05** Order rational numbers and place them on the number line. **Understand rational numbers and their location on the number line** **N.ME.06.17** Locate negative rational numbers (including integers) on the number line; know that numbers and their negatives add to 0, and are on opposites sides and at equal distance from 0 on a number line.		**Understand real number concepts** **N.ME.08.03** Understand that in decimal form, rational numbers either terminate or eventually repeat, and that calculators truncate or round repeating decimals; locate rational numbers on the number line; know fraction forms of common repeating decimals, e.g. $0.1 = \frac{1}{9}$; $0.3 = \frac{1}{3}$. **Understand real number concepts** **N.ME.08.04** Understand that irrational numbers are those that cannot be expressed as the quotient of two integers, and cannot be represented by terminating or repeating decimals; approximate the position of familiar irrational numbers, e.g. $\sqrt{2}$, $\sqrt{3}$, on the number line.
2. Express numerical comparisons as ratios and rates.	**Express, interpret, and use ratios; find equivalences** **N.ME.05.23** Express ratios in several ways, given applied situations, e.g., 3 cups to 5 people, 3:5, $\frac{3}{5}$; recognize and find equivalent ratios.	**Find equivalent ratios** **N.ME.06.11** Find equivalent ratios by scaling up or scaling down. **Calculate rates** **A.PA.06.01** Solve applied problems involving rates, including speed, e.g., if a car is going 50 mph, how far will it go in $3\frac{1}{2}$ hours?	**Understand derived quantities** **N.ME.07.01** Understand derived quantities such as density, velocity, and weighted averages. **Understand and solve problems involving rates, ratios, and proportions** **N.MR.07.04** Convert ratio quantities between different systems of units, such as feet per second to miles per hour. **Understand and apply directly proportional relationships; relate to linear relationships** **A.PA.07.01** Recognize when information in a table, graph or formula suggests a proportional or linear relationship. **A.RP.07.02** Represent directly proportional and linear relationships using verbal descriptions, tables, graphs and formulas, and translate among these representations. **A.PA.07.03** Given a directly proportional or linear situation, graph and interpret the slope and intercept(s) in terms of the original situation; evaluate $y = kx$ for specific x values, given k, e.g., weight vs. volume of water, base cost plus cost per unit. **A.PA.07.05** Understand and use directly proportional relationships of the form $y = mx$, and distinguish from linear relationships of the form $y = mx + b$, b non-zero; understand that in a directly proportional relationship between two quantities one quantity is a constant multiple of the other quantity. **Understand and represent linear functions:** **A.PA.07.06** Calculate the slope from the graph of a linear function as the ratio of "rise/run" for a pair of points on the graph, and express the answer as a fraction and a decimal; understand that linear functions have slope that is a constant rate of change. **A.PA.07.07** Represent linear functions in the form $y = x + b$, $y = mx$, and $y = mx + b$, and graph, interpreting slope and y-intercept.	**Solve problems** **N.MR.08.07** Understand percent increase and percent decrease in both sum and product form, e.g. 3% increase of a quantity x is $x + .03x = 1.03x$ **Understand the concept of non-linear functions using basic examples** **A.PA.08.02** For basic functions, e.g. simple quadratics, direct and indirect variation, and population growth, describe how changes in one variable affect the others.

3. Distinguish between prime and composite numbers; identify factors, multiples, common factors and multiples, and relatively prime numbers; and apply divisibility tests to numbers.	**Find prime factorizations of whole numbers** **N.MR.05.07** Find the prime factorization of numbers between 1 and 50, express in exponential notation, e.g., $24 = 2^3 \times 3^1$, and understand that every whole number can be expressed as a product of primes.			**Understand real number concepts** **N.ME.08.01** Understand the meaning of a square root of a number and its connection to the square whose area is the number; understand the meaning of a cube root and its connection to the volume of a cube. **N.ME.08.02** Understand meanings for zero and negative integer exponents.
4. Explain the meaning of powers and roots of numbers and use calculators to compute powers and square roots.	**Multiply and divide by powers of ten** **N.MR.05.15** Multiply a whole number by powers of 10: 0.01, 0.1, 1, 10, 100, and 1000; identify patterns.	**Use exponents** **N.ME.06.16** Understand and use integer exponents, excluding powers of negative numbers; express numbers in scientific notation.	**Recognize irrational numbers** **N.MR.07.06** Understand the concept of square root and cube root, and estimate using calculators.	**Understand the concept of non-linear functions using basic examples** **A.RP.08.01** Identify and represent linear functions, quadratic functions, and other simple functions including inverse functions ($y = k/x$), cubics ($y = ax^3$), roots ($y = \sqrt{x}$), and exponentials ($y = a^x$, $a > 0$), using tables, graphs, and equations.
5. Apply their understanding of number relationships in solving problems.			**Understand derived quantities** **N.FL.07.02** Solve problems involving derived quantities. **Understand and solve problems involving rates, ratios, and proportions** **N.FL.07.03** Calculate rates of change, including speed. **N.FL.07.05** Solve simple proportion problems using such methods as unit rate, scaling, finding equivalent fractions, and solving the proportion equation $a/b = c/d$; know how to see patterns about proportional situations in tables. **Understand and apply directly proportional relationships; relate to linear relationships** **A.PA.07.04** For directly proportional or linear situations, solve applied problems using graphs and equations; e.g., the heights and volume of a container with uniform cross-section; height of water in a tank being filled at a constant rate; degrees Celsius and degrees Fahrenheit; distance and time under constant speed.	**Solve problems** **N.FL.08.09** Solve problems involving compounded interest or multiple discounts. **N.MR.08.10** Calculate weighted averages such as course grades, consumer price indices, and sports ratings. **N.MR.08.11** Solve problems involving ratio units, such as miles per hour, dollars per pound, or persons per square mile.